The wavelets: A notion to study non-differentiable continuous functions

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The most natural tools to study the pointwise regularity of a function are the notions of continuity and derivability. However, these notions are not precise enough. Indeed, it is well-known that the function $f_h : x \mapsto |x|^h$ ($h \in (0,1]$) is non-differentiable and continuous at the origin; moreover, the smaller h is, the "less regular" the graph of f seems to be around 0. The value of the exponent h gives an information about the pointwise regularity of f_h at the origin and seems "to be a transition" between the continuity and the derivability.

To formalise this intuition, we will introduce the *Hölder spaces* $\Lambda^h(x_0)$, which verify the following property:

$$C^1(x_0) \subset \Lambda^{h_2}(x_0) \subset \Lambda^{h_1}(x_0) \subset C^0(x_0), \quad (0 < h_1 < h_2 < 1).$$

Moreover, for a long time, mathematicians thought that any continuous function is differentiable except on a set of isolated points. The first published example (1872) of a function continuous everywhere and nowhere differentiable is the *Weierstraß function*. In this talk, to prove that it is nowhere differentiable, we will introduce the notion of *wavelets*, which is a powerful tool to study this kind of functions [2].

To conclude the presentation, we will briefly present a recent algorithm based on the wavelets, which allows to study irregular signals: the *Leaders Profile Method* (LPM) [1]. This method is being used to study the topography of the planet Mars and the first results will be presented.

References

[1] C. Esser, T. Kleyntssens, and S. Nicolay. A multifractal formalism for non-concave and non-increasing spectra: The leaders profile method. *Applied and Computational Harmonic Analysis*, 43, 269–291, 2017.

[2] S. Jaffard. Wavelet techniques in multifractal analysis, Fractal Geometry and Applications: A Jubilee of Benoit Mandelbrot. *Proceedings of Symposia in Pure Mathematics*, 72, 91–151, 2004.

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